Redundancy Optimization of Trusses against Uncertainty in Structural Damage

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theme

- robust optimization of structures
  - a lot of studies
- redundancy optimization of structures **(!)**
  - very limited
    - application of redundancy in coding theory (Shannon) to truss
      [Mohr, Stein, Matzies, & Knapek '14]
    - cond. prob. of failure of component (given structural failure)
      gap between max. & min. $\rightarrow$ Min. [Mousavi & Gardoni '14]
  - redundancy: amount of damage that a structure
    can sustain w/o losing its functionality $\rightarrow$ Max. **(!)**
- Y. K.: "Redundancy optimization of finite-dimensional structures: a concept and a derivative-free algorithm." *J. Struct. Eng. (ASCE)*, to appear.

measures of structural redundancy

- degree of static determinacy $s = m - \operatorname{rank} H$
  - m : # of members (bars)
  - H : equilibrium matrix
    ($H^\top$: compatibility matrix; rigidity matrix)

measures of structural redundancy

- degree of static determinacy $s = m - \operatorname{rank} H$
- strength redundancy factor $r = l_{\text{intact}}/(l_{\text{intact}} - l_{\text{damaged}})$

  [Frangopol & Curley '87]

  - l_{intact} : ultimate strength of the intact structure
  - l_{damaged} : ultimate strength of the damaged structure

measures of structural redundancy

- degree of static determinacy $s = m - \operatorname{rank} H$
- strength redundancy factor $r = l_{\text{intact}}/(l_{\text{intact}} - l_{\text{damaged}})$

[Frangopol & Curley '87]

  - l_{intact} : ultimate strength of the intact structure
  - l_{damaged} : ultimate strength of the damaged structure
- $(P(D) - P(C))/P(C)$ [Fu & Frangopol '90]
  - $P(C)$: prob. of system collapse
  - $P(D)$: prob. of failure of a structural component

measures of structural redundancy

- degree of static determinacy $s = m - \operatorname{rank} H$
- strength redundancy factor $r = l_{\text{intact}}/(l_{\text{intact}} - l_{\text{damaged}})$

  [Frangopol & Curley '87]
- $(P(D) - P(C))/P(C)$ [Fu & Frangopol '90]
- residual strength index l_i/l_{u} [Feng & Moses '86]
  - l_{u} : ultimate strength
  - l_i : strength after the ith structural component has failed

measures of structural redundancy

- degree of static determinacy $s = m - \operatorname{rank} H$
- strength redundancy factor $r = l_{\mathrm{intact}}/(l_{\mathrm{intact}} - l_{\mathrm{damaged}})$

  [Frangopol & Curley '87]
- $(P(D) - P(C))/P(C)$ [Fu & Frangopol '90]
- residual strength index l_i/l_{u} [Feng & Moses '86]
- redundancy-strength index $l_{\mathrm{u}}/l_{\mathrm{y}}$ [Husain & Tsopelas '04]
  - l_{u} : ultimate strength
  - l_{y} : strength at "the first significant yielding (damage of a component)"

measures of structural redundancy

- degree of static determinacy $s = m - \operatorname{rank} H$
- strength redundancy factor $r = l_{\text{intact}}/(l_{\text{intact}} - l_{\text{damaged}})$
  [Frangopol & Curley '87]
- $(P(D) - P(C))/P(C)$ [Fu & Frangopol '90]
- residual strength index l_i/l_{u} [Feng & Moses '86]
- redundancy-strength index $l_{\text{u}}/l_{\text{y}}$ [Husain & Tsopelas '04]
- strong redundancy [K. & Ben-Haim '11]

measures of structural redundancy

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- strong redundancy [K. & Ben-Haim '11]

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common understanding

- high redundancy:
  small degradation of performance
  when some structural components are damaged

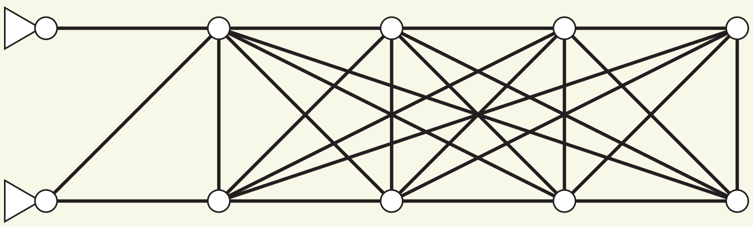
redundancy measure

- strong redundancy
  - := greatest level of structural degradation
    without violating the performance requirement

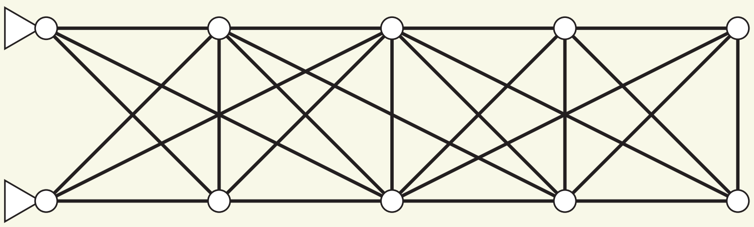
[K. & Ben-Haim ’11]

redundancy measure

- strong redundancy
- Which has higher redundancy?



(A)

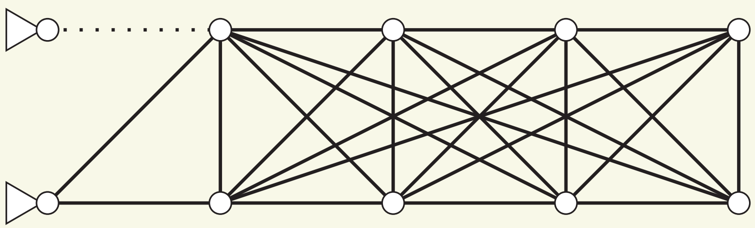


(B)

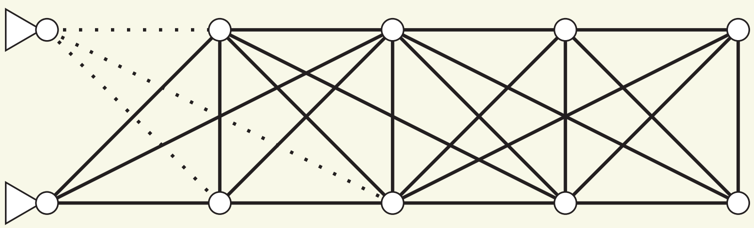
  - # of members $= 25$
  - deg. of static indeterminacy $= 9$

redundancy measure

- strong redundancy
- Which has higher redundancy?
  - Concerning stability (rigidity) requirement, (A) $<$ (B), because



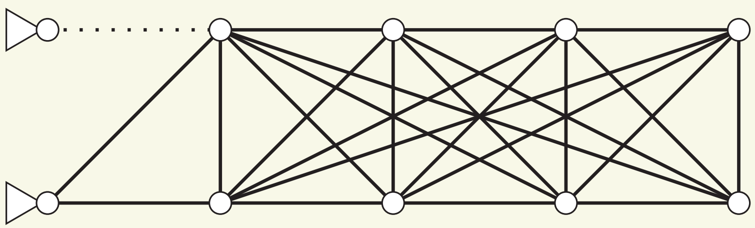
(A′)
(s. r.) $= 0$



(B′)
(s. r.) $= 2$

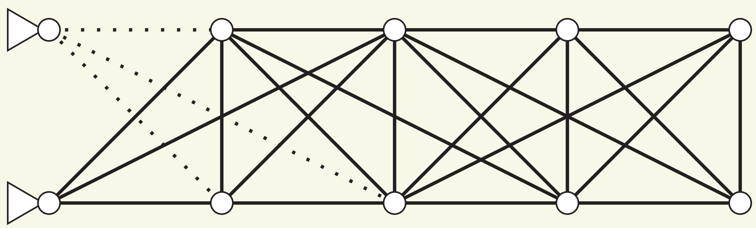
redundancy measure

- strong redundancy
- Which has higher redundancy?
  - Concerning stability (rigidity) requirement, (A) $\prec$ (B), because



(A′)

(s. r.) $= 0$



(B′)

(s. r.) $= 2$

  - (A′) and (B′) are the worst damage scenarios.

- a design problem:
  - maximizing redundancy

redundancy optimization: a concept

redundancy optimization: a concept

- deficiency set $\mathcal{D}(\alpha)$:

  set of damage scenarios s. t. at most α bars are absent

- structural performance $p(\boldsymbol{x})$

- performance in the worst scenario:

$$p^{\rm ws}(\alpha) := \min\{p(\boldsymbol{x}) \mid \boldsymbol{x} \in \mathcal{D}(\alpha)\}$$

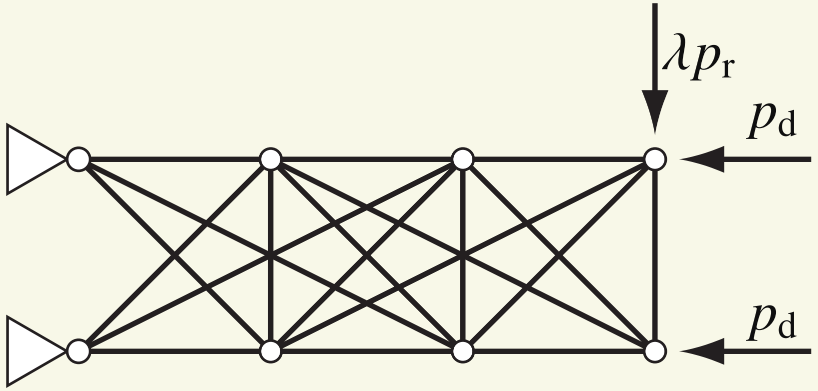
- redundancy optimization:

$$\text{Maximize} \quad p^{\rm ws}(\alpha) \qquad (\alpha : \text{ given})$$

  - uncertainty: damage scenario
    - optimize: performance in the worst case
    - (one of) difficulties: dependency of the worst case on design variables

redundancy optimization: an example

- deficiency set $\mathcal{D}(\alpha)$:
  set of damage scenarios s. t. at most α bars are absent
- structural performance: limit load factor λ^*
  - conventional optimization: maximizing λ^*



redundancy optimization: an example

- deficiency set $\mathcal{D}(\alpha)$:
  set of damage scenarios s. t. at most α bars are absent
- structural performance: limit load factor λ^*
  - conventional optimization: maximizing λ^*
- limit load factor in the worst scenario

$$\lambda^{\mathrm{ws}}(\alpha) := \min\{\lambda^*(\boldsymbol{x}) \mid \boldsymbol{x} \in \mathcal{D}(\alpha)\}$$

  - can be computed via mixed-integer programming. [K. '12]
- redundancy optimization: maximizing $\lambda^{\mathrm{ws}}(\alpha)$
  - design variables: bar cross-sectional areas
  - constraint: total bar volume

an algorithm: overview

an algorithm: overview

- difficulty of the optimization problem
  - For a given design:
    - evaluation of objective function requires expensive simulation
      — limit analysis (convex optimization)
    - no analytical gradient
- derivative-free method
  - stencil gradient [Kelly '99]
    - $\simeq$ finite difference method, but
    - (usu.) less # of function evaluations
- SQP (sequential quadratic programming)
  - use stencil gradient as a substitute of gradient

an algorithm: stencil gradient

- current point: $\boldsymbol{x}$ sample points: $z_1, \dots, z_n$
- stencil gradient:

$$\nabla_{\mathrm{s}} f(\boldsymbol{x}) := Y^{+} \boldsymbol{\delta}$$

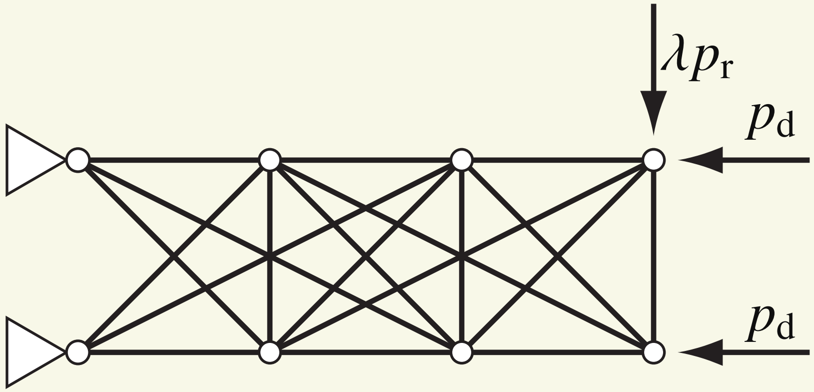
$$Y = \left[z_1 - \boldsymbol{x}_1 \;\middle|\; \cdots \;\middle|\; z_n - \boldsymbol{x}_n \right], \quad \boldsymbol{\delta} = \begin{bmatrix} f(z_1) - f(\boldsymbol{x}) \\ \vdots \\ f(z_n) - f(\boldsymbol{x}) \end{bmatrix}$$

  - special case: $Y = tI$ → finite difference

- previous iterations $\boldsymbol{x}_{k-1}, \boldsymbol{x}_{k-2}, \dots$ can be used as sample points
  → (usu.) less # of function evaluations
- implicit filtering (adaptive change of difference increment) [Kelly '99]
  - large → small as optim. proceeds
  - may possibly avoid poor local minimum

optimization results

- problem setting



optimization results

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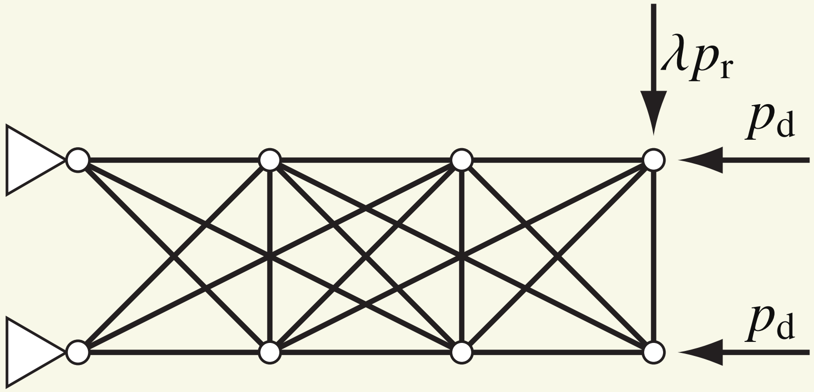
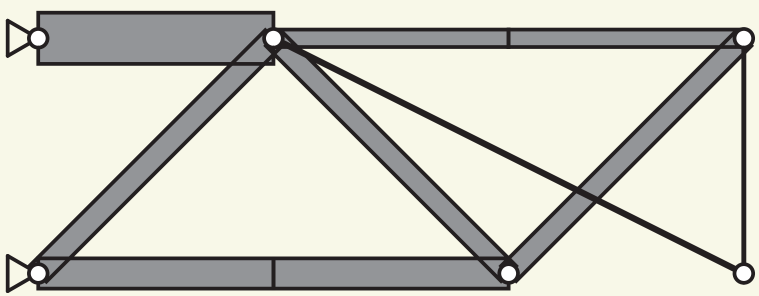
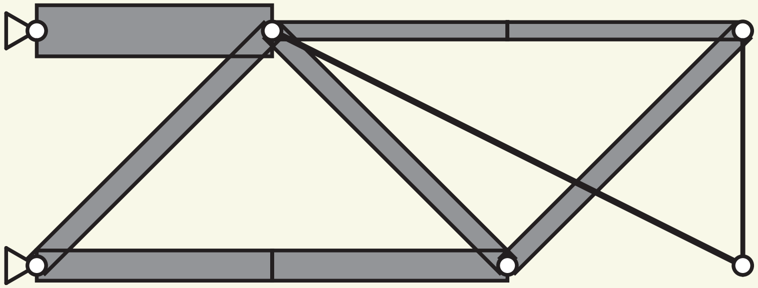


- conventional optimization (w/o considering redundancy)
  - maximize limit load factor λ^{*}
  - design variables: bar areas



optimization results: conventional optimization

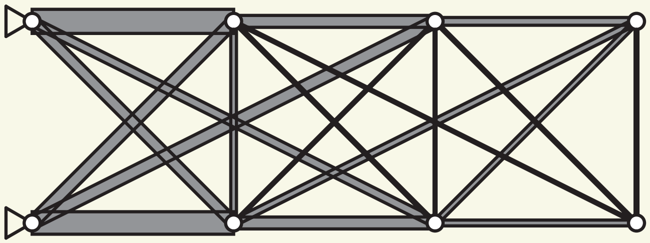
- conventional optimization (w/o considering redundancy)



  - By removing one bar, the solution becomes unstable.
    - deg. of static determinacy $= 0$ (i.e., statically determinate)
    - redundancy measure $= 0$

optimization results: redundancy optimization

- maximize λ^* w/ redundancy measure $\alpha = 1$,
  - i.e., optim. against the worst scenario s. t. one bar is missing.



  - 376 SQP iters.
  - 3699 MILP for worst-case analysis

optimization results: redundancy optimization

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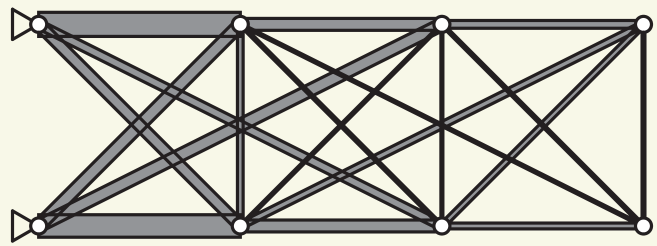
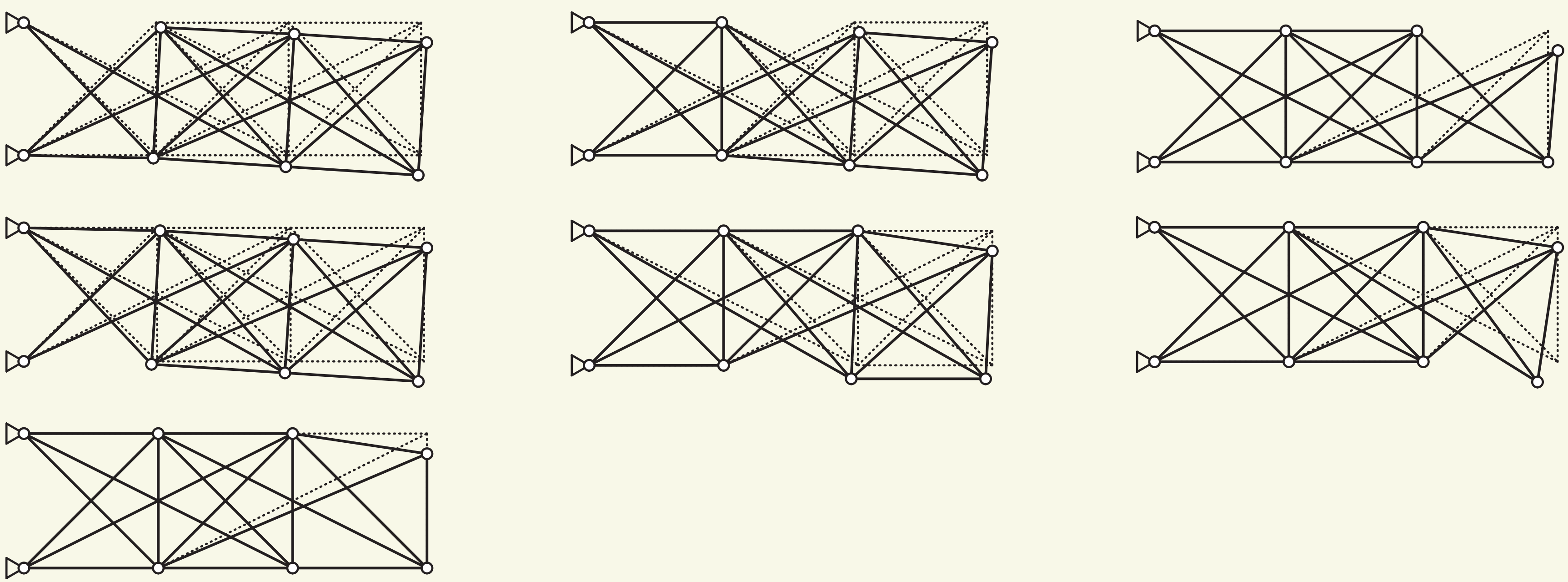


- multiple worst scenarios at the opt. sol.



optimization results: redundancy optimization

- maximize λ^{*} w/ redundancy measure $\alpha = 1$,
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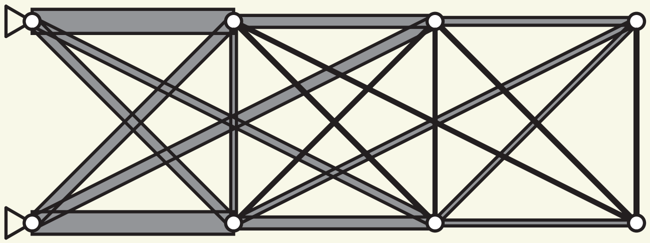
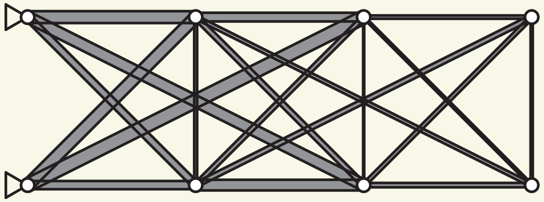


- multiple worst scenarios at the opt. sol.
- multiplicity of w. s. means
  redundancy optim. is nonsmooth optim.

optimization results: redundancy optimization

- maximize λ^{*} w/ redundancy measure $\alpha = 2$,
  - i.e., optim. against the worst scenario s. t. at most two bars can possibly be missing.



  - 327 SQP iters.
  - 3326 MILPs

optimization results: redundancy optimization

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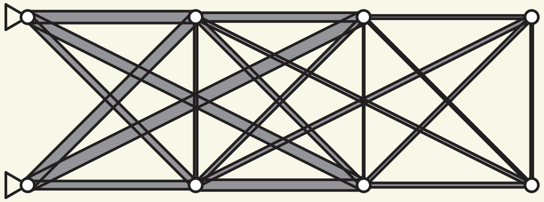

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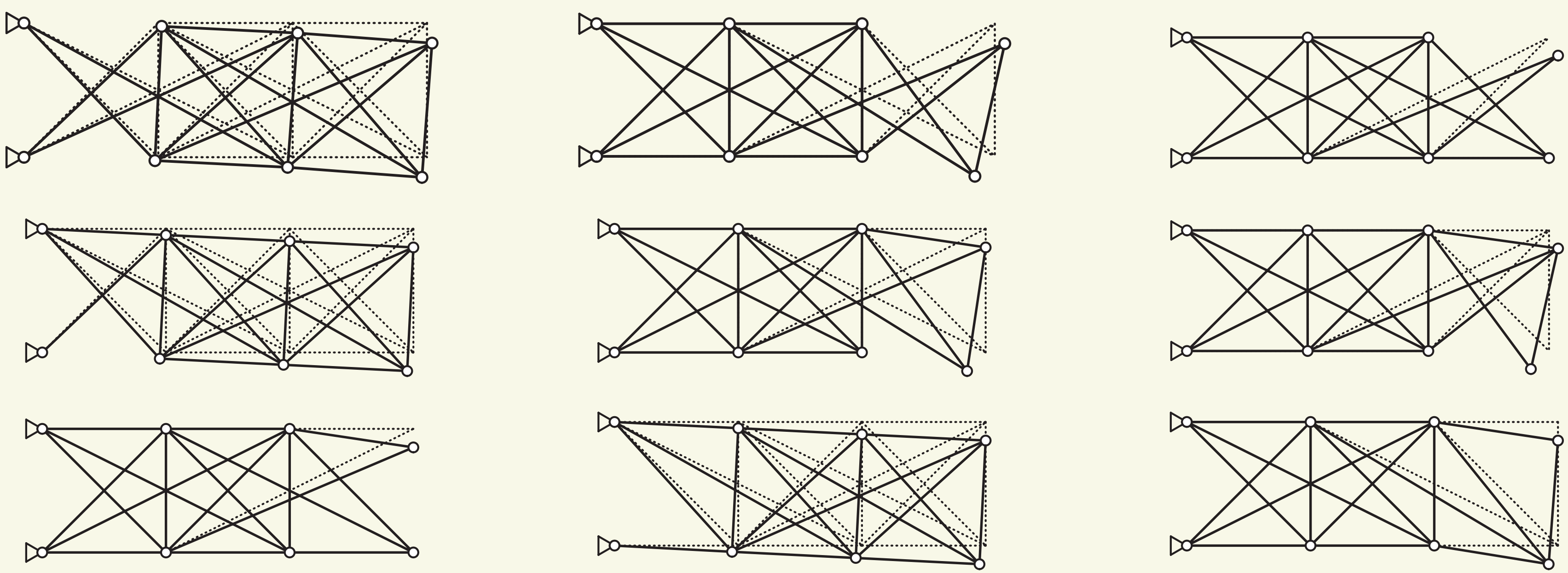
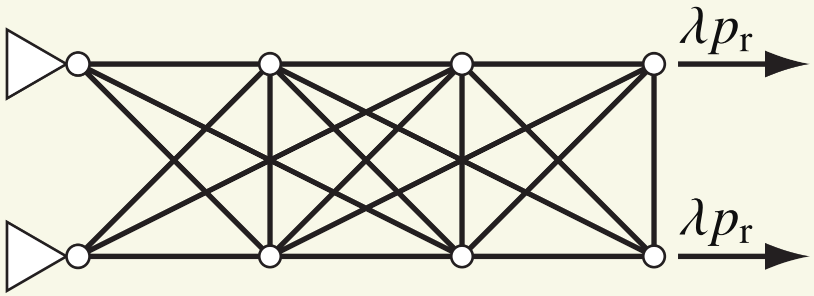
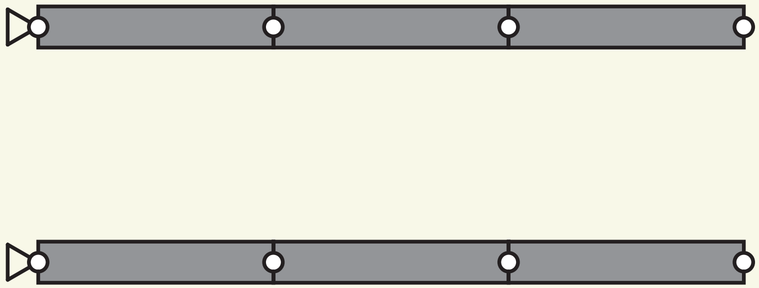


another example

- problem setting



- conventional optimization (w/o considering redundancy)
  - maximize limit load factor λ^*
  - design variables: bar areas



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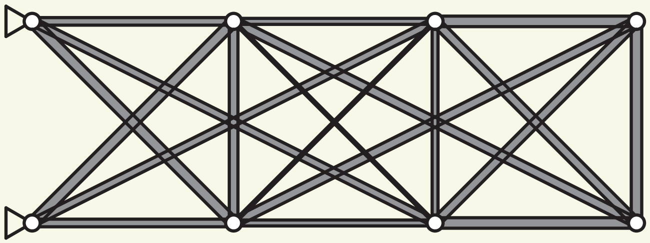
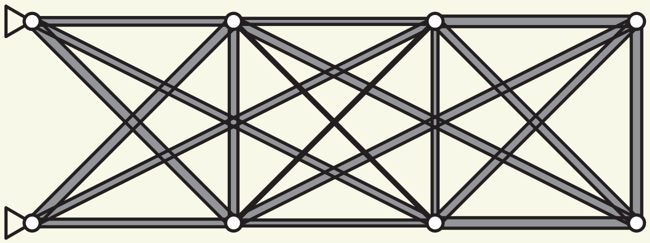


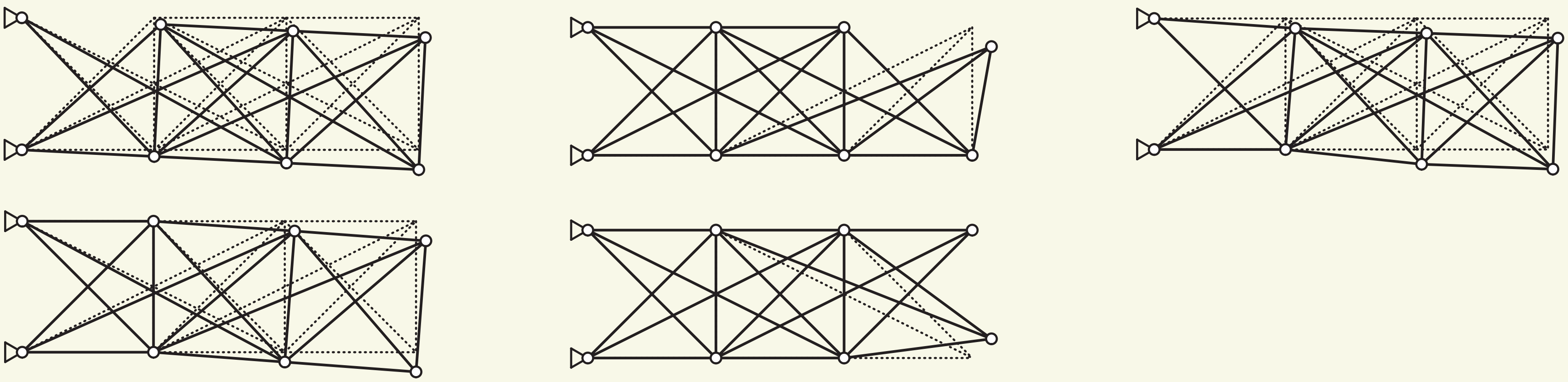
  - 200 SQP iters.
  - 1960 MILPs

another example: redundancy optimization

- maximize λ^* w/ redundancy measure $\alpha = 1$,
  - i.e., optim. against the worst scenario s. t. one bar is missing.



- multiple worst scenarios at the opt. sol.



  - multiplicity $= 9$

another example: redundancy optimization

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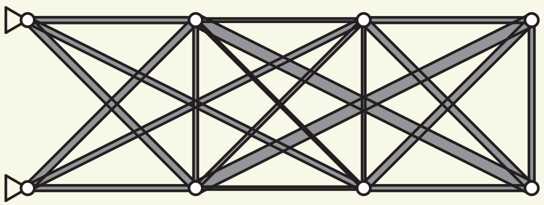

  - 378 SQP iters.
  - 4014 MILPs

another example: redundancy optimization

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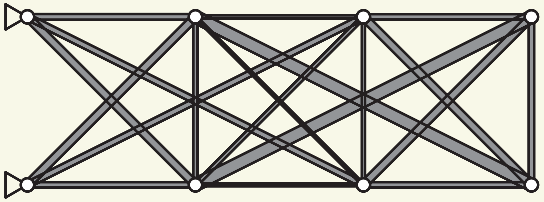
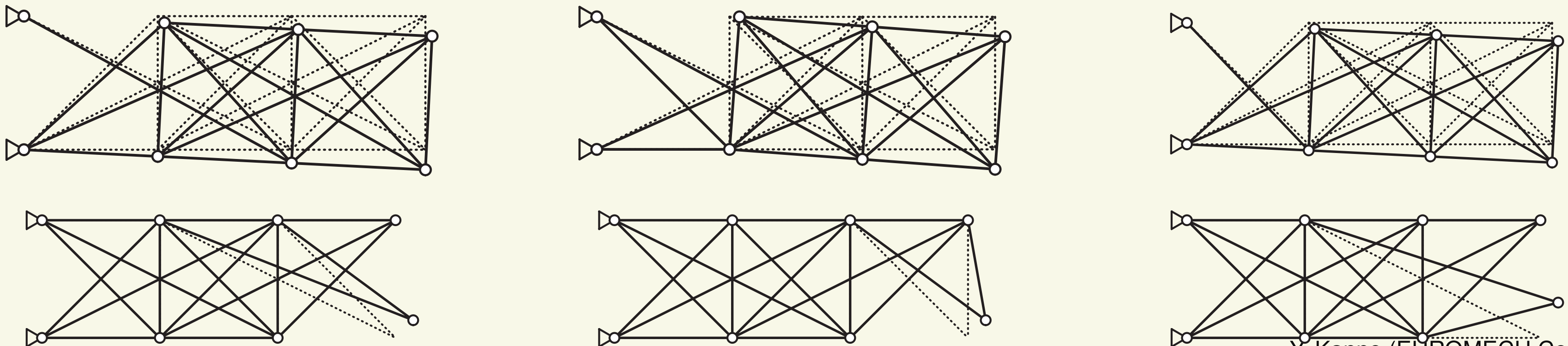


  - 378 SQP iters.
  - 4014 MILPs

- multiple worst scenarios at the opt. sol. (multiplicity $= 18$)



conclusions

- structural redundancy
  - greatest level of structural degradation
    without violating the performance requirement
- worst scenario in limit analysis
  - given: # of damaged structural components
    $\sim$ redundancy measure
  - uncertainty: damage
- redundancy optimization
  - maximize the limit load factor in the worst scenario
  - multiplicity of worst scenarios at an optimal solution